

[p. 14] Exemple $\circ y'' + y = \frac{1}{\cos t}$

Sol. (H) $y'' + y = 0$. $r^2 + 1 = 0$ (C) $r = \pm i$

$y_{\text{re}}(t) = A \cos t + B \sin t$

Sol. particulière: $y(t) = A(t) \cos t + B(t) \sin t$

$$\begin{cases} A'(t)y_1(t) + B'(t)y_2(t) = 0 \\ A'y_1' + B'y_2' = \frac{f(x)}{a} \end{cases} \quad \begin{cases} A'(t) \cos t + B'(t) \sin t = 0 \\ -A'(t) \sin t + B'(t) \cos t = \frac{1}{\cos t} \end{cases}$$

On trouve: $A'(t) = -B'(t) \frac{\sin t}{\cos t}$

$$B'(t) \frac{\sin^2 t}{\cos t} + B'(t) \cos t = \frac{1}{\cos t}$$

$$B'(t) \sin^2 t + B'(t) \cos^2 t = 1$$

$$B'(t) = 1 \Rightarrow B(t) = t + C_1, \quad C_1 = \text{const}$$

$$(\ln x)' = \frac{1}{x} \quad (\ln(u(x)))' = \frac{u'(x)}{u(x)}$$

$$A'(t) = -\frac{\sin t}{\cos t} = (\ln(\cos t))' \Rightarrow A(t) = \ln(\cos t) + C_2$$

$t \in]-\frac{\pi}{2}, \frac{\pi}{2}[$

Sol. part.: $y_{\text{part}} = \ln(\cos t) \cos t + t \sin t$

Solution: $y(t) = (A + \ln(\cos t)) \cos t + (B + t) \sin t, \quad A, B \in \mathbb{R}$