MATH103_MISPI
TD du chapitre 8 : Intégrales et primitives

L1 MISPI

Questions de cours

1. Soit f une fonction continue sur un intervalle I, $a \in I$, et $g : x \mapsto \int_a^x f(t)\,dt$. Que vaut $g'(x)$?
2. Soit f une fonction continue sur un intervalle $[a, b]$.
   a. Que vaut la valeur moyenne de f sur $[a, b]$?
   b. Donner la formule de l'inégalité de la moyenne.
3. Soit f une fonction continue sur un intervalle $[a, b]$. Que peut-on écrire à propos de $\left|\int_a^b f(t)\,dt\right|$?
4. Soit f une fonction continue sur $\mathbb{R}$ et T périodique. Compléter : $\int_a^{a+T} f(t)\,dt = \int_0^{\cdots} f(t)\,dt = \int_{-T/2}^{\cdots} f(t)\,dt$.

Exercice 2

Calculer les intégrales et primitives suivantes (reconnaître des dérivées) :

$$A = \int_1^3 \left(\frac{3}{x^2} + \frac{\sqrt{x}}{x^5} \right) dx \quad B = \int_1^2 \frac{3x^2 + 4x - 2 - \sqrt{x}}{x^4} dx \quad C = \int_0^1 (2x+3)(x^2+3x-5)^3 dx$$

$$D = \int 4e^{3t-1} dt \qquad E = \int \frac{3x}{(x^2+2)} dx \qquad F = \int_0^1 \frac{x}{\sqrt{4x^2+3}} dx$$

$$G = \int_0^1 x\sqrt{x^2+3}\, dx \qquad H = \int_0^1 \frac{3x}{(x^2+2)^2} dx \qquad I = \int_{\pi/3}^{\pi/4} \tan x \, dx$$

Exercice 2 suite

A

$$A = \int_1^3 \left(\frac{3}{x^2} + \frac{\sqrt{x}}{x^5} \right) \, dx = \left[-3\frac{1}{x} \right]_1^3 + \left[-\frac{2}{7}\frac{1}{x^{7/2}} \right]_1^3$$

$$= -3(\frac{1}{3} - 1) - \frac{2}{7}(3^{-\frac{7}{2}} - 1) = 2 + \frac{2}{7} - \frac{2}{7}3^{-\frac{7}{2}}$$

$$= \frac{16}{7} - \frac{2}{7}3^{-3}\sqrt{3}^{-1} = \frac{16}{7} - \frac{2}{189\sqrt{3}} \simeq 2,28.$$

Exercice 2 suite

A

$$A = \int_1^3 \left(\frac{3}{x^2} + \frac{\sqrt{x}}{x^5} \right) \, dx = \left[-3\frac{1}{x} \right]_1^3 + \left[-\frac{2}{7}\frac{1}{x^{7/2}} \right]_1^3$$

$$= -3(\frac{1}{3} - 1) - \frac{2}{7}(3^{-\frac{7}{2}} - 1) = 2 + \frac{2}{7} - \frac{2}{7}3^{-\frac{7}{2}}$$

$$= \frac{16}{7} - \frac{2}{7}3^{-3}\sqrt{3}^{-1} = \frac{16}{7} - \frac{2}{189\sqrt{3}} \simeq 2,28.$$

B

$$B = \int_1^2 \frac{3x^2 + 4x - 2 - \sqrt{x}}{x^4} \, dx = \left[\frac{-3}{x} + \frac{-2}{x^2} + \frac{2}{3x^3} + \frac{2}{5}\frac{1}{x^{5/2}} \right]_1^2$$

$$= -\frac{3}{2} - \frac{1}{2} + \frac{1}{12} + \frac{2}{5}\frac{1}{2^{5/2}} - (-3 - 2 + \frac{2}{3} + \frac{2}{5})$$

$$= 3 - \frac{2}{3} - \frac{2}{5} + \frac{1}{12} + \frac{1}{10\sqrt{2}} = \frac{121}{60} + \frac{1}{10\sqrt{2}} \simeq 2,09.$$

- c

$$C = \int_0^1 (2x+3)(x^2+3x-5)^3 \, dx = \left[\frac{1}{4}(x^2+3x-5)^4\right]_0^1$$
$$= \frac{1}{4}\left((1+3-5)^4 - (-5)^4\right) = \frac{1}{4}((-1)^4 - (-5)^4) = -\frac{624}{4} = -156.$$

Exercice 2 suite

- C

$$C = \int_0^1 (2x+3)(x^2+3x-5)^3 \, dx = \left[\frac{1}{4}(x^2+3x-5)^4\right]_0^1$$
$$= \frac{1}{4}\left((1+3-5)^4 - (-5)^4\right) = \frac{1}{4}((-1)^4 - (-5)^4) = -\frac{624}{4} = -156.$$

- D $D = \displaystyle\int 4e^{3t-1} \, dt = \frac{4}{3}e^{3t-1} + k$ où $k \in \mathbb{R}$.

- C

$$C = \int_0^1 (2x+3)(x^2+3x-5)^3 \, dx = \left[\frac{1}{4}(x^2+3x-5)^4\right]_0^1$$
$$= \frac{1}{4}\left((1+3-5)^4 - (-5)^4\right) = \frac{1}{4}((-1)^4 - (-5)^4) = -\frac{624}{4} = -156.$$

- D $D = \int 4e^{3t-1} \, dt = \frac{4}{3}e^{3t-1} + k$ où $k \in \mathbb{R}$.

- E $E = \int \frac{3x}{(x^2+2)} \, dx = \frac{3}{2}\ln(|x^2+2|) + k = \frac{3}{2}\ln(x^2+2) + k$ où $k \in \mathbb{R}$.

- C

$$C = \int_0^1 (2x+3)(x^2+3x-5)^3\ dx = \left[\frac{1}{4}(x^2+3x-5)^4\right]_0^1$$
$$= \frac{1}{4}\left((1+3-5)^4 - (-5)^4\right) = \frac{1}{4}((-1)^4 - (-5)^4) = -\frac{624}{4} = -156.$$

- D $D = \displaystyle\int 4e^{3t-1}\ dt = \frac{4}{3}e^{3t-1} + k$ où $k \in \mathbb{R}$.
- E $E = \displaystyle\int \frac{3x}{(x^2+2)}\ dx = \frac{3}{2}\ln(|x^2+2|) + k = \frac{3}{2}\ln(x^2+2) + k$ où $k \in \mathbb{R}$.
- F $F = \displaystyle\int_0^1 \frac{x}{\sqrt{4x^2+3}}\ dx = \left[\frac{1}{4}\sqrt{4x^2+3}\right]_0^1 = \frac{1}{4}(\sqrt{7}-\sqrt{3}) \simeq 0,23.$

G

$$G = \int_0^1 x\sqrt{x^2+3}\,dx = \left[\frac{1}{3}\left(x^2+3\right)^{3/2}\right]_0^1 = \frac{1}{3}(4^{3/2} - 3^{3/2})$$
$$= \frac{1}{3}(4\sqrt{4} - 3\sqrt{3}) = \frac{8}{3} - \sqrt{3} \simeq 0,93.$$

Exercice 2 suite

- G

$$G = \int_0^1 x\sqrt{x^2+3}\,dx = \left[\frac{1}{3}\left(x^2+3\right)^{3/2}\right]_0^1 = \frac{1}{3}(4^{3/2} - 3^{3/2})$$
$$= \frac{1}{3}(4\sqrt{4} - 3\sqrt{3}) = \frac{8}{3} - \sqrt{3} \simeq 0,93.$$

- H $H = \int_0^1 \frac{3x}{(x^2+2)^2}\,dx = \left[-\frac{3}{2}\frac{1}{x^2+2}\right]_0^1 = \frac{1}{4}.$

- G

$$G = \int_0^1 x\sqrt{x^2+3}\,dx = \left[\frac{1}{3}\left(x^2+3\right)^{3/2}\right]_0^1 = \frac{1}{3}(4^{3/2}-3^{3/2})$$
$$= \frac{1}{3}(4\sqrt{4}-3\sqrt{3}) = \frac{8}{3}-\sqrt{3} \simeq 0,93.$$

- H $H = \int_0^1 \frac{3x}{\left(x^2+2\right)^2}\,dx = \left[-\frac{3}{2}\frac{1}{x^2+2}\right]_0^1 = \frac{1}{4}.$

- I

$$I = \int_{\pi/3}^{\pi/4} \tan x\ dx = -\int_{\pi/3}^{\pi/4} \frac{-\sin x}{\cos x}\,dx = \Big[-\ln|\cos(x)|\Big]_{\pi/3}^{\pi/4}$$
$$= \Big[\ln|\cos(x)|\Big]_{\pi/4}^{\pi/3} = \ln\left(\frac{\frac{1}{2}}{\frac{\sqrt{2}}{2}}\right) = -\frac{\ln 2}{2} \simeq -0,35.$$

Exercice 3

Calculer les intégrales et primitives suivantes en utilisant une ou plusieurs intégrations par parties :

$$A = \int_{-\pi}^{\pi} x\sin(3x)\,dx \quad B(x) = \int x\ln(x)\,dx$$

$$C = \int_{0}^{\ln 3} (x-1)e^{-2x}\,dx \quad D(x) = \int x^2 2^x\,dx$$

Par intégration par parties, on obtient :

A

$$A = \int_{-\pi}^{\pi} x\sin(3x)\,dx \overset{x\mapsto x\sin(3x)\ \text{paire}}{=} 2\int_0^{\pi} x\sin(3x)\,dx$$

I.P.P : $u(x) = x\,,\ u'(x) = 1\,;\ v'(x) = \sin(3x)\,,\ v(x) = \dfrac{-\cos(3x)}{3}$

$$= 2\left(\left[-\frac{x\cos(3x)}{3}\right]_0^{\pi} + \frac{1}{3}\int_0^{\pi}\cos(3x)\,dx\right)$$

$$= 2\left(\frac{\pi}{3} + \left[\frac{1}{9}\sin(3x)\right]_0^{\pi}\right) = \frac{2\pi}{3} + 0 = \frac{2\pi}{3} \simeq 2,09.$$

Par intégration par parties, on obtient :

A

$$A = \int_{-\pi}^{\pi} x\sin(3x)\,dx \overset{x\mapsto x\sin(3x)\ \text{paire}}{=} 2\int_0^{\pi} x\sin(3x)\,dx$$

I.P.P : $u(x) = x\,,\ u'(x) = 1\,;\ v'(x) = \sin(3x)\,,\ v(x) = \dfrac{-\cos(3x)}{3}$

$$= 2\left(\left[-\frac{x\cos(3x)}{3}\right]_0^{\pi} + \frac{1}{3}\int_0^{\pi}\cos(3x)\,dx\right)$$

$$= 2\left(\frac{\pi}{3} + \left[\frac{1}{9}\sin(3x)\right]_0^{\pi}\right) = \frac{2\pi}{3} + 0 = \frac{2\pi}{3} \simeq 2,09.$$

B I.P.P : $u(x) = \ln(x)\,,\ u'(x) = \dfrac{1}{x}\,;\ v'(x) = x\,,\ v(x) = \dfrac{x^2}{2}$

$$B = \int x\ln(x)\,dx = \frac{x^2\ln x}{2} - \frac{1}{2}\int x\,dx = \frac{x^2\ln x}{2} - \frac{x^2}{4} + k \text{ où } k\in\mathbb{R}.$$

- I.P.P : $u(x) = (x-1)$, $u'(x) = 1$; $v'(x) = e^{-2x}$, $v(x) = \dfrac{e^{-2x}}{-2}$

$$\begin{aligned}
C &= \int_0^{\ln 3} (x-1)e^{-2x}\, dx = \left[-\frac{(x-1)e^{-2x}}{2} \right]_0^{\ln 3} + \frac{1}{2}\int_0^{\ln 3} e^{-2x}\, dx \\
&= -\frac{1}{2}\left(\frac{\ln 3 - 1}{3^2} + 1 \right) + \frac{1}{2}\left[-\frac{1}{2} e^{-2x} \right]_0^{\ln 3} = -\frac{\ln 3 + 8}{18} - \frac{1}{4}(\frac{1}{9} - 1) \\
&= -\frac{\ln 3 + 8}{18} + \frac{2}{9} = -\left(\frac{\ln 3}{18} + \frac{2}{9} \right) \simeq -0,28.
\end{aligned}$$

C I.P.P : $u(x) = (x-1)$, $u'(x) = 1$; $v'(x) = e^{-2x}$, $v(x) = \dfrac{e^{-2x}}{-2}$

$$C = \int_0^{\ln 3} (x-1)e^{-2x}\,dx = \left[-\frac{(x-1)e^{-2x}}{2}\right]_0^{\ln 3} + \frac{1}{2}\int_0^{\ln 3} e^{-2x}\,dx$$
$$= -\frac{1}{2}\left(\frac{\ln 3 - 1}{3^2} + 1\right) + \frac{1}{2}\left[-\frac{1}{2}e^{-2x}\right]_0^{\ln 3} = -\frac{\ln 3 + 8}{18} - \frac{1}{4}\left(\frac{1}{9} - 1\right)$$
$$= -\frac{\ln 3 + 8}{18} + \frac{2}{9} = -\left(\frac{\ln 3}{18} + \frac{2}{9}\right) \simeq -0,28.$$

D I.P.P : $u(x) = x^2$, $u'(x) = 2x$; $v'(x) = 2^x$, $v(x) = \dfrac{2^x}{\ln(2)}$

$$D(x) = \int x^2 2^x\,dx = x^2\frac{2^x}{\ln 2} - \frac{2}{\ln 2}\int x2^x\,dx$$

I.P.P : $u(x) = x$, $u'(x) = 1$; $v'(x) = 2^x$, $v(x) = \dfrac{2^x}{\ln(2)}$

$$= x^2\frac{2^x}{\ln 2} - \frac{2}{\ln 2}\left(x\frac{2^x}{\ln 2} - \frac{1}{\ln 2}\int 2^x\,dx\right)$$
$$= \frac{2^x}{\ln 2}\left(x^2 - \frac{2}{\ln 2}x + \frac{2}{\ln^2 2}\right) + k \text{ où } k \in \mathbb{R}.$$

Exercice 4

1. Soit $a > 0\,,\ a \neq 1$. Calculer $A = \displaystyle\int_0^2 a^t \, dt$.
2. Calculer $B = \displaystyle\int_0^2 \sqrt{|x-1|}\, dx$ (indication : $|1-x| = \ldots \ \mathrm{si} \ldots$).
3. $C = \displaystyle\int \frac{\ln x}{x}\, dx \quad D = \int \frac{1}{x \ln x}\, dx \quad E = \int x^2 \ln(x)\, dx$
   $F = \displaystyle\int \frac{\sin(x)}{\cos^5(x)}\, dx \quad G = \int_0^{\pi/2} \big(\cos(t)\big)^3\, dt$. (*indication : on pourra exprimer* $(\cos t)^3$ *en fonction de* $\cos(3t)$ *et de* $\cos(t)$).

- Soit $a > 0$, $a \neq 1$. $A = \displaystyle\int_0^2 a^t \, dt = \left[\frac{a^t}{\ln a}\right]_0^2 = \frac{a^2 - 1}{\ln a}$.

- A Soit $a > 0\,,\ a \neq 1$. $A = \displaystyle\int_0^2 a^t\ dt = \left[\frac{a^t}{\ln a}\right]_0^2 = \frac{a^2-1}{\ln a}$.
- B

$$\begin{aligned} B = \int_0^2 \sqrt{|x-1|}\ dx &= \int_0^1 \sqrt{1-x}\ dx + \int_1^2 \sqrt{x-1}\ dx \\ &= -\frac{2}{3}\left[(1-x)^{3/2}\right]_0^1 + \frac{2}{3}\left[(x-1)^{3/2}\right]_1^2 = \frac{4}{3}. \end{aligned}$$

- A Soit $a > 0$, $a \neq 1$. $A = \displaystyle\int_0^2 a^t \, dt = \left[\frac{a^t}{\ln a}\right]_0^2 = \frac{a^2 - 1}{\ln a}$.
- B

$$B = \int_0^2 \sqrt{|x-1|}\, dx = \int_0^1 \sqrt{1-x}\, dx + \int_1^2 \sqrt{x-1}\, dx$$
$$= -\frac{2}{3}\left[(1-x)^{3/2}\right]_0^1 + \frac{2}{3}\left[(x-1)^{3/2}\right]_1^2 = \frac{4}{3}.$$

- C $C = \displaystyle\int \frac{\ln x}{x}\, dx = \frac{1}{2}\ln(x)^2 + k$ où $k \in \mathbb{R}$.

- A Soit $a > 0\,,\ a \neq 1$. $A = \displaystyle\int_0^2 a^t\,dt = \left[\frac{a^t}{\ln a}\right]_0^2 = \frac{a^2-1}{\ln a}$.
- B

$$B = \int_0^2 \sqrt{|x-1|}\,dx = \int_0^1 \sqrt{1-x}\,dx + \int_1^2 \sqrt{x-1}\,dx$$
$$= -\frac{2}{3}\left[(1-x)^{3/2}\right]_0^1 + \frac{2}{3}\left[(x-1)^{3/2}\right]_1^2 = \frac{4}{3}.$$

- C $C = \displaystyle\int \frac{\ln x}{x}\,dx = \frac{1}{2}\ln(x)^2 + k$ où $k \in \mathbb{R}$.
- D $D = \displaystyle\int \frac{1}{x\ln x}\,dx = \ln|\ln(x)| + k$ où $k \in \mathbb{R}$.

- A Soit $a > 0$, $a \neq 1$. $A = \int_0^2 a^t \, dt = \left[\frac{a^t}{\ln a}\right]_0^2 = \frac{a^2 - 1}{\ln a}$.
- B

$$B = \int_0^2 \sqrt{|x-1|}\, dx = \int_0^1 \sqrt{1-x}\, dx + \int_1^2 \sqrt{x-1}\, dx$$
$$= -\frac{2}{3}\left[(1-x)^{3/2}\right]_0^1 + \frac{2}{3}\left[(x-1)^{3/2}\right]_1^2 = \frac{4}{3}.$$

- C $C = \int \frac{\ln x}{x}\, dx = \frac{1}{2}\ln(x)^2 + k$ où $k \in \mathbb{R}$.
- D $D = \int \frac{1}{x \ln x}\, dx = \ln|\ln(x)| + k$ où $k \in \mathbb{R}$.
- E $E = \int x^2 \ln(x)\, dx \overset{IPP}{=} \frac{1}{3}\, x^3 \ln(x) - \frac{1}{3}\int x^2\, dx = \frac{1}{3}\, x^3 \ln(x) - \frac{1}{9}x^3 + k$ où $k \in \mathbb{R}$.

- A. Soit $a > 0$, $a \neq 1$. $A = \int_0^2 a^t \, dt = \left[\frac{a^t}{\ln a}\right]_0^2 = \frac{a^2 - 1}{\ln a}$.
- B.

$$B = \int_0^2 \sqrt{|x-1|} \, dx = \int_0^1 \sqrt{1-x} \, dx + \int_1^2 \sqrt{x-1} \, dx$$
$$= -\frac{2}{3}\left[(1-x)^{3/2}\right]_0^1 + \frac{2}{3}\left[(x-1)^{3/2}\right]_1^2 = \frac{4}{3}.$$

- C. $C = \int \frac{\ln x}{x} \, dx = \frac{1}{2} \ln(x)^2 + k$ où $k \in \mathbb{R}$.
- D. $D = \int \frac{1}{x \ln x} \, dx = \ln|\ln(x)| + k$ où $k \in \mathbb{R}$.
- E. $E = \int x^2 \ln(x) \, dx \overset{IPP}{=} \frac{1}{3} x^3 \ln(x) - \frac{1}{3} \int x^2 \, dx = \frac{1}{3} x^3 \ln(x) - \frac{1}{9} x^3 + k$ où $k \in \mathbb{R}$.
- F. $F = \int \frac{\sin(x)}{\cos^5(x)} \, dx = \frac{1}{4} \cdot \frac{1}{\cos^4(x)} + k$ où $k \in \mathbb{R}$.

- G $G = \int_0^{\pi/2} \big(\cos(t)\big)^3 \, dt$. On commence par linéariser $\big(\cos(t)\big)^3$. On a $\big(\cos(t)\big)^3 = \left(\dfrac{e^{it} + e^{-it}}{2}\right)^3 \overset{\text{Binôme}}{=} \dfrac{1}{8} \sum_{k=0}^{3} \binom{3}{k} e^{ikt} e^{-i(3-k)t}$. On obtient : $\big(\cos(t)\big)^3 = \dfrac{1}{4} \dfrac{e^{-3it} + 3e^{-it} + 3e^{it} + e^{3it}}{2} = \left(\dfrac{\cos(3t)}{4} + \dfrac{3\cos(t)}{4}\right)$. Ainsi :

$$G = \int_0^{\pi/2} \left(\frac{\cos(3t)}{4} + \frac{3\cos(t)}{4}\right) dt = \left[\frac{\sin(3t)}{12} + \frac{3\sin(t)}{4}\right]_0^{\pi/2}$$
$$= -\frac{1}{12} + \frac{3}{4} = \frac{8}{12} = \frac{2}{3}.$$

(*On a aussi*

$\cos^3 = \cos^2 \cdot \cos = (1 - \sin^2)\cos = \cos - \cos\sin^2 = \sin' - (\frac{1}{3} \cdot \sin^3)'.$)

Exercice 5

Calculer les intégrales et primitives suivantes en utilisant le changement de variable indiqué :

$$A = \int_0^1 \frac{2e^x}{e^x + e^{-x}} dx \quad (t = e^x)$$

$$B = \int_1^2 t^3 e^{t^2}\, dt \quad (v = t^2)$$

$$C(x) = \int \frac{3}{2(3-x)^2 + 2}\, dx \quad (t = (3-x))$$

$$D(x) = \int \frac{x^2}{\sqrt{2x+1}}\, dx \quad (t = \sqrt{2x+1})$$

- A $A = \int_0^1 \frac{2e^x}{e^x + e^{-x}} dx$. On pose $t = e^x$ ou $x = \ln t$ donc $dx = \frac{dt}{t}$. Doù

$$A = \int_1^e \frac{2t}{t + \frac{1}{t}} \frac{dt}{t} = \int_1^e \frac{2t}{t^2 + 1} dt = \Big[\ln(t^2 + 1) \Big]_1^e = \ln(1 + e^2) - \ln 2.$$

A. $A = \displaystyle\int_0^1 \frac{2e^x}{e^x + e^{-x}} dx$. On pose $t = e^x$ ou $x = \ln t$ donc $dx = \dfrac{dt}{t}$. Doù

$$A = \int_1^e \frac{2t}{t + \frac{1}{t}} \frac{dt}{t} = \int_1^e \frac{2t}{t^2+1} dt = \left[\ln(t^2+1) \right]_1^e = \ln(1+e^2) - \ln 2.$$

B. $B = \displaystyle\int_1^2 t^3 e^{t^2}\, dt$. On pose $v = t^2$ ou $t = \sqrt{v}$ donc $dt = \dfrac{dv}{2\sqrt{v}}$. D'où

$$B = \int_1^4 \left(\sqrt{v}\right)^3 e^v \frac{dv}{2\sqrt{v}} = \frac{1}{2}\int_1^4 ve^v dv \overset{IPP}{=} \frac{1}{2}\left(\left[ve^v\right]_1^4 - \int_1^4 e^v dv \right)$$

$$= \frac{1}{2}\left(4e^4 - e - \left[e^v\right]_1^4\right) = \frac{1}{2}(4e^4 - e - (e^4 - e)) = \frac{3}{2}e^4.$$

- **c** $C(x) = \int \frac{3}{2(3-x)^2+2}\,dx$. On pose $t = 3 - x$ donc $dt = -dx$. D'où

$$\begin{aligned} C(x) &= \int \frac{3}{2t^2+2}(-dt) = -\frac{3}{2}\int \frac{dt}{t^2+1} = -\frac{3}{2}\arctan t + k \\ &= \frac{3}{2}\arctan(x-3) + k \end{aligned}$$

avec $k \in \mathbb{R}$.

- $C(x) = \displaystyle\int \frac{3}{2(3-x)^2+2}\, dx$. On pose $t = 3 - x$ donc $dt = -dx$. D'où

$$C(x) = \int \frac{3}{2t^2+2}(-dt) = -\frac{3}{2}\int \frac{dt}{t^2+1} = -\frac{3}{2}\arctan t + k$$
$$= \frac{3}{2}\arctan(x-3) + k$$

avec $k \in \mathbb{R}$.

- $D(x) = \displaystyle\int \frac{x^2}{\sqrt{2x+1}}\, dx$. On pose $t = \sqrt{2x+1}$ ou $x = \dfrac{1}{2}(t^2-1)$ donc $dx = t\ dt$. D'où

$$D(x) = \int \frac{\frac{1}{4}(t^2-1)^2}{t} t\ dt = \frac{1}{4}\int (t^2-1)^2 dt = \frac{1}{4}(\frac{t^5}{5} - \frac{2}{3}\ t^3 + t) + k$$
$$= \frac{\sqrt{2x+1}}{4}\left(\frac{1}{5}(2x+1)^2 - \frac{2}{3}(2x+1) + 1\right) + k \text{ où } k \in \mathbb{R}$$

- D (suite)

$$D(x) = \frac{\sqrt{2x+1}}{4}\left(\frac{4}{5}x^2 - \frac{8}{15}x + \frac{8}{15}\right) + k$$
$$= \frac{\sqrt{2x+1}}{5}\left(x^2 - \frac{2}{3}x + \frac{2}{3}\right) + k.$$